Hemispherical dynamos generated by convection in rotating spherical shells

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At Taylor numbers of the order 10^8 and Prandtl numbers of the order 1, dynamos generated by convection are found for which the magnetic field is essentially confined to either the northern or the southern hemisphere of a rotating spherical shell. The time dependence is typically chaotic, but latitudinal waves can be discerned that cause the magnetic field to change its polarity in a cyclical fashion. A possible relationship to a solar phenomenon is pointed out.

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The dynamo process in which magnetic fields are generated by motions in an electrically conducting fluid is generally believed to be the cause of large-scale magnetic fields observed in the universe. Stellar and planetary magnetic fields in particular are generated by convection flows in the electrically conducting interiors of stars and planets. The problem of convection driven dynamos in rotating spherical shells is thus of fundamental importance for numerous applications in geophysics and astrophysics. The increase in recent years of readily available computer capacity has permitted the exploration of regions of the parameter space for spherical dynamos that were not accessible hitherto. In this paper we wish to draw attention to the phenomenon of hemispherical dynamos.

In the following we use a standard formulation of the spherical dynamo problem with a minimum number of physical parameters that include those of primary importance for stellar and planetary applications. While it is not possible to reach in numerical simulations the asymptotically high values of the dimensionless parameters realized in cosmic bodies, it seems feasible to attain appropriate values of parameters such as the Rayleigh number and the Taylor number if the latter are based on eddy diffusivities instead of molecular diffusivities.

We consider a spherical fluid shell of thickness d that is rotating about a fixed axis with the constant angular velocity Ω. We assume that a static state exists with the temperature distribution $T_S=T_0-\beta d^2r^2/2$ and that the gravity field is given by $\vec{g}=-\gamma d\vec{r}$ where $\vec{r}$ is the position vector with respect to the center of the sphere and r is its length measured in units of d. In addition to the length d, the time d^2/ν, the temperature $\beta d^2\nu/\kappa$, and the magnetic flux density $\nu(\mu\varrho)^{1/2}/d$ are used as scales for the dimensionless description of the problem where ν denotes the kinematic viscosity of the fluid, κ its thermal diffusivity, μ is the magnetic permeability, and ϱ its density. The latter is assumed to be constant except in the gravity term where its temperature dependence given by $\alpha\equiv(d\varrho/dT)/\varrho=$ const is taken into account. Since the velocity field $\vec{u}$ as well as the magnetic flux density $\vec{B}$ are solenoidal vector fields, we can use the general representation in terms of poloidal and toroidal components,

$$\vec{u}=\vec{\nabla}\times(\vec{\nabla}v\times\vec{r})+\vec{\nabla}w\times\vec{r}, \tag{1a}$$

$$\vec{B}=\vec{\nabla}\times(\vec{\nabla}h\times\vec{r})+\vec{\nabla}g\times\vec{r}. \tag{1b}$$

By multiplying the $(\mathrm{curl})^2$ and the curl of the Navier-Stokes equations in the rotating system by $\vec{r}$ we obtain two equations for v and w,

$$[(\nabla^2-\partial_t)L_2+\tau\partial_\varphi]\nabla^2v+\tau Qw-RL_2\Theta = -\vec{r}\cdot\vec{\nabla}\times[\vec{\nabla}\times(\vec{u}\cdot\vec{\nabla}\vec{u}-\vec{B}\cdot\vec{\nabla}\vec{B})], \tag{2a}$$

$$[(\nabla^2-\partial_t)L_2+\tau\partial_\varphi]w-\tau Qv=\vec{r}\cdot\vec{\nabla}\times(\vec{u}\cdot\vec{\nabla}\vec{u}-\vec{B}\cdot\vec{\nabla}\vec{B}), \tag{2b}$$

where ∂_t and ∂_φ denote the partial derivatives with respect to time t and with respect to the angle φ of a spherical system of coordinates r,θ,φ and where the operators L_2 and Q are defined by

$$L_2\equiv-r^2\nabla^2+\partial_r(r^2\partial_r),$$

$$Q\equiv r\cos\theta\nabla^2-(L_2+r\partial_r)(\cos\theta\partial_r-r^{-1}\sin\theta\partial_\theta).$$

The heat equation for the dimensionless deviation Θ from the static temperature distribution can be written in the form

$$\nabla^2\Theta+L_2v=P(\partial_t+\vec{u}\cdot\vec{\nabla})\Theta \tag{2c}$$

and the equations for h and g are obtained through the multiplication of the equation of magnetic induction and of its curl by $\vec{r}$,

$$\nabla^2L_2h=P_m[\partial_tL_2h-\vec{r}\cdot\vec{\nabla}\times(\vec{u}\times\vec{B})], \tag{2d}$$

$$\nabla^2L_2g=P_m[\partial_tL_2g-\vec{r}\cdot\vec{\nabla}\times\{\vec{\nabla}\times(\vec{u}\times\vec{B})\}]. \tag{2e}$$

The Rayleigh number R, the Taylor number τ^2, the Prandtl number P, and the magnetic Prandtl number P_m are defined by

$$R=\frac{\alpha\gamma\beta d^6}{\nu\kappa},\quad \tau=\frac{2\Omega d^2}{\nu},\quad P=\frac{\nu}{\kappa},\quad P_m=\frac{\nu}{\lambda}, \tag{3}$$

where λ is the magnetic diffusivity. We assume stress-free boundaries with fixed temperatures,

$$v = \partial^2_{rr} v = \partial_r(w/r) = \Theta = 0 \text{ at } r = r_i \equiv \eta/(1-\eta)$$
$$\text{and at } r = r_o = (1-\eta)^{-1}, \tag{4a}$$

where η is the radius ratio, $\eta = r_i/r_o$. Throughout this paper only the case $\eta = 0.4$ will be considered. For the magnetic field electrically insulating boundaries are used such that the poloidal function h must be matched to the function $h^{(e)}$ that describes the potential fields outside the fluid shell

$$g = h - h^{(e)} = \partial_r(h - h^{(e)}) = 0 \text{ at } r = r_i \text{ and } r = r_o \tag{4b}$$

The numerical integration of Eqs. (2) together with boundary conditions (4) proceeds with the pseudospectral method described in [1] that is based on an expansion of all dependent variables in spherical harmonics for the θ, φ dependences, i.e.,

$$v = \sum_{l,m} V_l^m(r,t) P_l^m(\cos\theta) \exp\{im\varphi\}, \tag{5}$$

and analogous expressions for the other variables, w, Θ, h, and g. P_l^m denotes the associated Legendre functions. For the r dependence, expansions in Chebychev polynomials are used. For further details see also [2].

For most of the computations to be reported, in the following, 33 collocation points in the radial direction and spherical harmonics up to the order 64 have been used.

Solutions of Eqs. (2) describing convection flows in the absence of a magnetic field are usually symmetric with respect to the equatorial plane if τ is sufficiently high, i.e., the contributions of spherical harmonics with odd $l-m$ in expression (5) vanish. Note that Θ has always the same symmetry as v, while w has the opposite symmetry in that spherical harmonic contributions with even $l-m$ vanish in the case of equatorial symmetry. Even in the case of high Rayleigh numbers R, when the time dependence of convection has become aperiodic, the equatorial symmetry persists if τ is high enough [3,1].

Because of this symmetry, solutions with a finite magnetic field bifurcating from the solution of nonmagnetic convection are expected to exhibit either a quadrupolar or a dipolar symmetry, i.e., the spherical harmonic contributions in the expression for h vanish either for odd $l-m$ or for even $l-m$. As in the case of w the symmetry of the toroidal field g is always opposite to that of h, i.e., for quadrupolar fields, contributions with even $l-m$ vanish while those with odd $l-m$ vanish for dipolar fields.

Predominantly dipolar magnetic fields are found in computations based on Eqs. (2), but also quadrupolar dynamos can be obtained in certain regions of the parameter space [4,5]. It thus was registered with surprise when dynamo computations for parameter values in the neighborhood of $\tau = 10^4$, $R = 5\times10^5$, $P = 1$, $P_m = 5$ exhibited hemispherical dynamos as shown in Fig. 1. Both quadrupolar and dipolar components contribute nearly equal magnetic energy in this case such that the contributions nearly cancel in one hemisphere while they are in phase in the other hemisphere. This phenomenon occurs at finite amplitudes of the magnetic field in a regime of aperiodic time dependence such that there is no contradiction with the symmetry consideration applicable at the bifurcation point. It is remarkable that the convection flow has lost little of its equatorial symmetry in the presence of the hemispherical magnetic field.

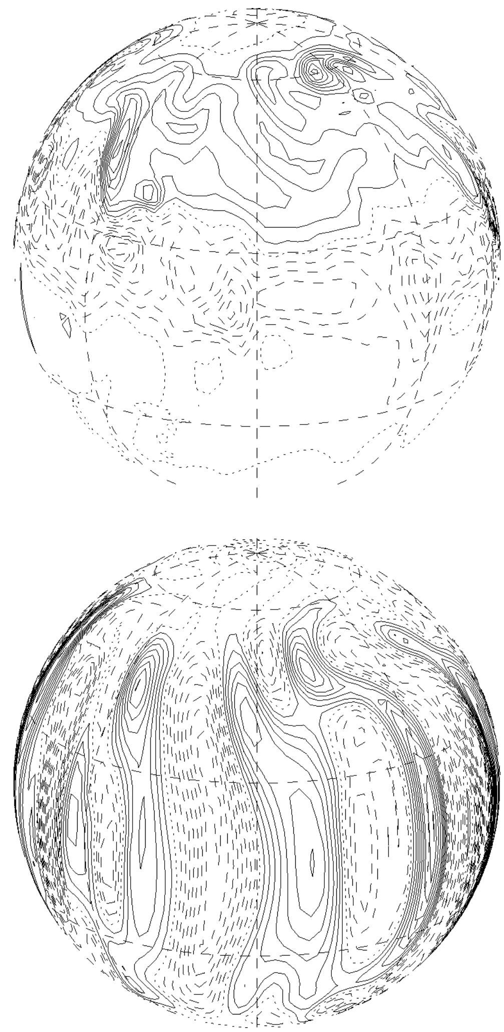

FIG. 1. Lines of constant radial component B_r of the magnetic field at the surface of the sphere, $r = r_o$ (upper plot) and lines of constant radial velocity u_r at the middle surface $r = (r_i + r_o)/2$ (lower plot) in the case $\tau = 10^4$, $R = 6.5\times10^5$, $P = 1$, $P_m = 2$. Solid (dashed) lines indicate positive (negative) values.

The original computations of hemispherical dynamos reported in [4] are misleading because a longitudinal symmetry had been employed in that only even values of m in expression (5) were included in the computations in order to save computer time. While this reduction of the full set of expansion functions did not affect in any significant way the quadrupolar and dipolar dynamos which has been tested in [4], the rather strong sensitivity of hemispherical dynamos to disturbances with the wave number $m = 1$ was found only more recently. Hence all computations reported in the present paper are based on the full set of expansion functions. Chaotic hemispherical dynamos can still be obtained when all wave numbers m are taken into account. But they are usually only found close to the lower limit of values of R for which dynamos can be obtained in the regime between the occurrence of quadrupolar and dipolar dynamos. With increasing R the nearly perfect correlation between dipolar and quadrupolar components tends to decay and the magnetic field assumes a more random appearance. Like quadrupolar dynamos [5]

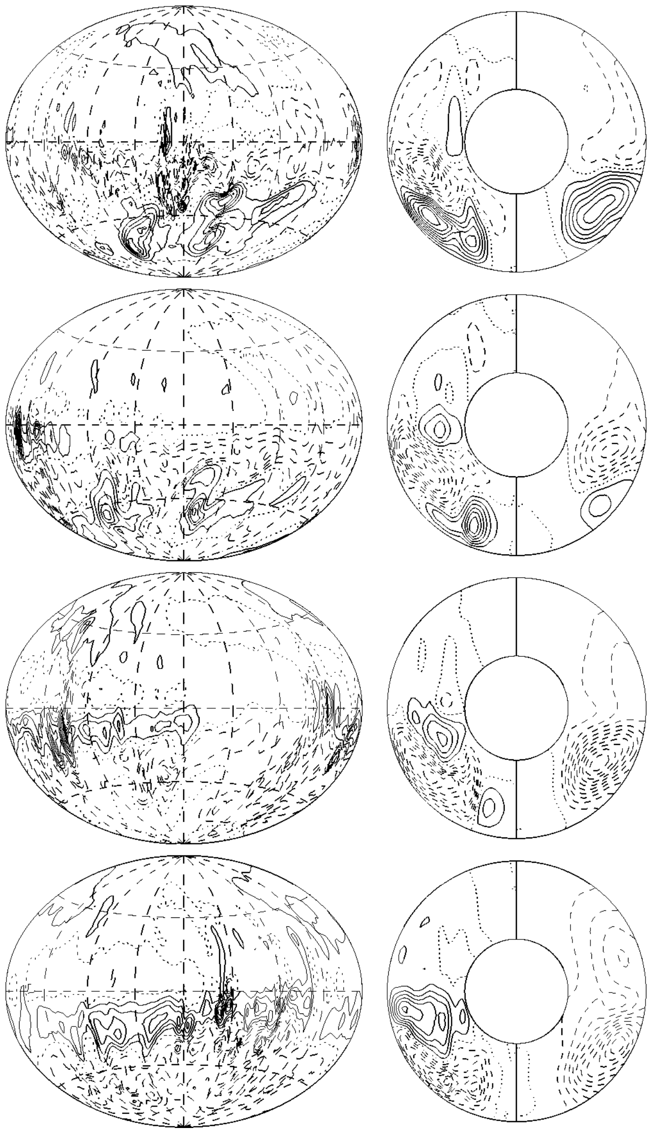

FIG. 2. Time sequence of plots covering approximately half a cycle of the hemispherical dynamo at the times $t=t_0+0.021n$, $n=0,1,2,3$ (from top to bottom) in the case $\tau=1.5\times10^4$, $R=11\times10^5$, $P=0.5$, $P_m=0.8$. The left plots show lines of constant B_r at the surface $r=r_o$. The right plots show lines of constant $\overline{B}_\varphi$ (left half) and field lines of $\vec{B}$ in the meridional plane (right half). The overbar indicates the axisymmetric component.

hemispherical dynamos usually exhibit a cyclical behavior corresponding to a magnetic field wave propagating to higher latitudes as shown in Fig. 2. This dynamo wave is most clearly seen in the mean azimuthal component of the field.

Since hemispherical dynamos occur in close competition with quadrupolar dynamos on the one side and with dipolar dynamos on the other side, intermittent transitions from one type to the other can often be observed as shown in the time plots of magnetic energies shown in Fig. 3. For the time interval $2.4 \lesssim t \lesssim 3.4$ the dipolar component of the magnetic field has nearly disappeared in this case. The energies plotted in this figure are defined by

$$\overline{E}_p=\frac{1}{2}\langle|\vec{\nabla}\times(\vec{\nabla}\overline{v}\times\vec{r})|^2\rangle, \quad \overline{E}_t=\frac{1}{2}\langle|\vec{\nabla}\overline{w}\times\vec{r}|^2\rangle, \tag{6a}$$

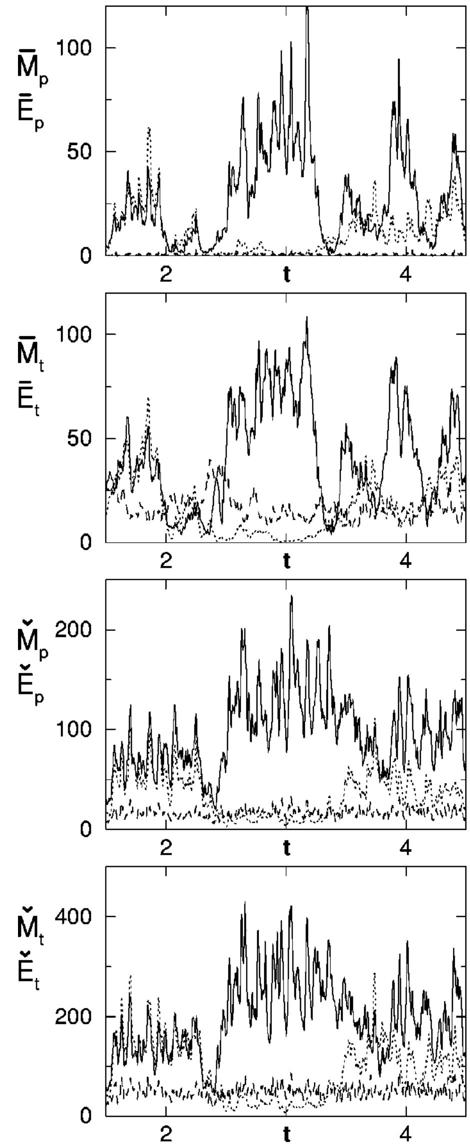

FIG. 3. Energy densities $\overline{M}_p$, $\overline{M}_t$, $\check{M}_p$, $\check{M}_t$ for quadrupolar (solid lines) and dipolar (dotted lines) components of the magnetic field as a function of time t in the case $\tau=10^4$, $R=6.5\times10^5$, $P=1$, $P_m=2$. Also shown are the kinetic energy densities $\overline{E}_p$, $\overline{E}_t$, $\check{E}_p$, $\check{E}_t$ (indicated by dashed lines). The scales for $\overline{E}_p$, $\overline{E}_t$, $\check{E}_p$, $\check{E}_t$ must be multiplied by the factors 0.2, 5, 10, 10, respectively.

$$\check{E}_p=\frac{1}{2}\langle|\vec{\nabla}\times(\vec{\nabla}\check{v}\times\vec{r})|^2\rangle, \quad \check{E}_t=\frac{1}{2}\langle|\vec{\nabla}\check{w}\times\vec{r}|^2\rangle, \tag{6b}$$

where the brackets $\langle\cdots\rangle$ indicate the average over the spherical shell, $\overline{v}$ denotes the axisymmetric component of v and $\check{v}\equiv v-\overline{v}$ denotes the azimuthally fluctuating component of v. The corresponding magnetic energies $\overline{M}_p, \overline{M}_t, \check{M}_p, \check{M}_t$ are defined analogously with h and g replacing v and w. In the case of the magnetic energies we have further separated them into dipolar and quadrupolar parts.

The computations shown in Fig. 3 have been extended until the time $t=7$. No further change in the character of the dynamo did occur except for a short interval of quadrupolar symmetry around $t=5.4$. Generally quadrupolar dynamos are preferred at somewhat higher values of P and slightly lower values of P_m. At magnetic Prandtl numbers of the order 10 or higher dipolar dynamos are preferred. It should be emphasized that in spite of their chaotic nature the three types of dynamos can clearly be distinguished as long as the symmetry of the velocity field with respect to the equatorial plane is approximately satisfied.

The basic reason for the preference of hemispherical dynamos seems to be that the Lorentz force acting on the convection columns is twice as effective for a given amount of generated magnetic flux when this flux is concentrated in one hemisphere instead of being distributed over both. In the case of chaotic dipolar and quadrupolar dynamos a localization of magnetic flux can also be observed. But it occurs in longitude rather than in latitude.

There is little evidence for hemispherical dynamos in stars or planets, except for one intriguing set of observations. During the Maunder sunspot minimum in the 17th century sunspots were observed almost exclusively in the southern hemisphere of the sun. Even when sunspots reappeared in substantial numbers in the cycle of 1700 to 1710, more than 95% of them occurred at southern latitudes [6]. There is thus a strong indication that the solar dynamo has operated in a hemispherical fashion during that period of time.

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